# STRESS STATE OF AN ELASTIC PLANE WEAKENED BY AN INFINITE SERIES OF LONGITUDINAL–TRANSVERSE CRACKS

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Use of the fact that a singular operator transforms a polynomial again into a polynomial permitted obtaining substantially new results in [1], devoted to wing theory. This property of singular operators is used to solve the plane problem of elasticity theory for a plane weakened by cracks. The criterion for the beginning of crack growth is related in the linear theory of fracture to the stress-intensity factor at its end. An investigation of the influence of the mutual arrangement of cracks on the intensity factor is of considerable interest. The intensity factor is zero in the stretching of a plane weakened by a longitudinal slit, but this factor grows in the presence of a transverse slit and may even exceed the intensity factor at the end of the transverse slit. In this case stratification of the material, the development of cracks located along the loading line, starts. Fractures of this kind have been observed in experiments. To solve the problem of determining the stress-intensity factor at the end of a longitudinal crack in the presence of a transverse crack, the consideration of a periodic system of longitudinal–transverse cracks turns out to be effective. Introduction of symmetry simplifies the construction of the solution of the problem, on the one hand, and is a good approximation to the problem of the mutual influence of two cracks for a sufficient mutual removal of the slits, on the other.

1. Let a plane weakened by the following two periodic systems of slits be given (Fig. 1). The longitudinal slits are directed along the line y = 0, have identical length 2c, and are located in intervals of length 2b (c < b) so that their middles x = 2kb (k = 0, ±1, ±2, ...) coincide with the centers of the intervals. The transverse slits, perpendicular to the line y = 0, have the identical length 2*a* with middles located at the points

$$x=b_k=(2k+1)\ b,\ y=0.$$

Given at infinity are the stress

$$\sigma_x^\infty = \sigma_1, \quad \sigma_y^\infty = \sigma_2, \quad \tau_{xy} = \tau,$$

and the edges of the slits are stress-free.

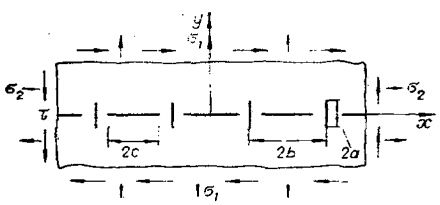


Fig. 1

The problem reduces to an equation for the stress function

$$\Delta^2 U=0,\ \Delta=(\ \ )_{xx}+(\ \ )_{yy} \tag{1.1}$$

under the conditions

$$U_{yy}=\sigma_1,\ U_{xx}=\sigma_2,\ U_{xy}=-\tau, \tag{1.2}$$

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at infinity,

$$L' = \sum_{k=-\infty}^{\infty} L'_k, \quad L'_k = \{2kb - c \leqslant x \leqslant 2kb + c, \quad y = 0\}$$
$$\overline{U}_{xx} = \overline{U}_{xy} = 0 \tag{1.3}$$

on the edges of the slits on the line $L'' = \sum_{k=-\infty}^{\infty} L''_k, \quad L''_k = \{x = b_k, \quad -a \leqslant y \leqslant a\}$ , and

$$U_{yy} = U_{xy} = 0. \tag{1.4}$$

on the line

We seek the solution of the problem in the form

$$U(x, y) = U^{\infty}(x, y) + U_1(x, y) + \sum_{k=-\infty}^{\infty} U_{2,k}(x, y). \tag{1.5}$$

where

$$U^{\infty}(x, y) = \frac{1}{2}(\sigma_1 y^2 + \sigma_2 x^2) - \tau xy; \tag{1.6}$$

$$U_1(x, y) = \frac{1}{2\pi} \int_{L'} [f_1(\xi)(x - \xi) + f_2(\xi) y] \ln[(x - \xi)^2 + y^2] \, d\xi; \tag{1.7}$$

$$U_{2,k}(x, y) = \frac{1}{2\pi} \int_{L''_k} [f_3(\eta)(x - b_k) + f_4(\eta)(y - \eta)] \ \ln[(x - b_k)^2 + (y - \eta)^2] \, d\eta. \tag{1.8}$$

The function (1.6) assures compliance with condition (1.2), while the form of the representations (1.7) and (1.8) is indicated by the application of the Fourier transform, as has been done in [2], so that the following representation of the solution of (1.1) results:

$$U(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [f_1(\xi)(x - \xi) + f_2(\xi) y] \ln[(x - \xi)^2 + y^2] \, d\xi.$$

Requiring that the functions $f_1(\xi)$, $f_2(\xi)$ vanish outside the line L', including the ends of its component segments $L'_k$, we obtain (1.7). The function $U_1(x, y)$ introduces singularities associated with weakening of the plane by longitudinal slits into the general solution of the problem. The representation (1.8) can be obtained analogously, with the sole difference that integration here is over the line $x = b_k = (2k + 1)b$ between the limits $-\infty < y < \infty$ and the functions $f_3(\eta)$, $f_4(\eta)$ are assumed zero outside the segment $L''_k$ and on its ends. The functions $U_{2,k}(x, y)$ introduce singularities associated with the transverse slits into the general solution of the problem.

The conditions (1.3) and (1.4) at the slit edges for the derivative functions (1.5) result in a system of four singular integral equations:

$$\int_{L'} \frac{f_1 + (x - \xi) f'_1}{x - \xi} d\xi + \sum_{k=-\infty}^{\infty} G_{2,k} \{(x - b_k)(2f_3 + \eta f'_3) - \eta f_4 - \eta^2 f'_4\} = -\pi\sigma_2; \tag{1.9}$$

$$\int_{L'} \frac{f_2 d\xi}{x - \xi} + \sum_{k=-\infty}^{\infty} G_{2,k} \{-\eta f_3 + (x - b_k)^2 f'_3 - \eta(x - b_k) f'_4\} = \pi\tau; \tag{1.10}$$

$$\int\limits_{L_0} \frac{f_4+(y-\eta) f_4'}{y-\eta} d\eta + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} G_{2,k}^0 \{f_4+(y-\eta) f_4'(y-\eta)\} + G_1 \{(b-\xi)(f_1-yf_2')+2yf_2-(b-\xi)^2 f_1'\} = -\pi\sigma_1; \quad (1.11)$$

$$\int\limits_{L_0} \frac{f_3 d\eta}{y-\eta} + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} G_{2,k}^0 \{(y-\eta) f_3+(b-b_k)^2 f_3'\} + G_1 \{(b-\xi)(f_2+yf_1')+y^2 f_2'\} = \pi\tau, \quad (1.12)$$

where $f_j = f_j(\xi)$ for j = 1, 2; $f_j = f_j(\eta)$ for j = 3, 4;

$$G_1\{S\} = \int\limits_{L'} \frac{S(\xi, y)\, d\xi}{(b-\xi)^2+y^2},$$

$$G_{2,k}\{S(x)\} = \int\limits_{L_k} \frac{S_k(\eta, x)\, d\eta}{(x-b_k)^2+\eta^2},$$

$$G_{2,k}^0\{S\} = G_{2,k}\{S(b)\}.$$

Here $\sum_{k=-\infty}^{\infty} G_{2,k}^0 \{(b-b_k) f\} = 0.$

The members containing $c_0 = c/b$, $a_0 = a/b$ in powers not above the fourth will be retained later in solving the system (1.9)-(1.12).

It is possible to write ($\pi^2/4b^2 = \varepsilon$)

$$\int\limits_{L'} \frac{f(\xi)\, d\xi}{x-\xi} = \sum_{k=-\infty}^{\infty} \int\limits_{2kb-c}^{2kb+c} \frac{f(\xi)\, d\xi}{x-\xi} = \sum_{k=-\infty}^{\infty} \int\limits_{-c}^{c} \frac{f(\xi)\, d\xi}{x-\xi-2kb} =$$

$$= \int\limits_{-c}^{c} \frac{f(\xi)\, d\xi}{x-\xi} + \sum_{k=1}^{\infty} \int\limits_{-c}^{c} \frac{f(\xi)\, 2(x-\xi)\, d\xi}{(x-\xi)^2+4k^2b^2} = \int\limits_{-c}^{c} \left[\frac{1}{x-\xi} - (x-\xi)\left(\frac{\varepsilon}{3}+\frac{\varepsilon^2}{45}(x-\xi)^2\right)\right] f(\xi)\, d\xi + 0\,(c_0^6);$$

$$\int\limits_{L'} \frac{f(\xi)\, d\xi}{(b-\xi)^2+y^2} = \sum_{k=-\infty}^{\infty} \int\limits_{-c}^{c} \frac{f(\xi)\, d\xi}{[(2k-1)\, b+\xi]^2+y^2} = \int\limits_{-c}^{c} \left[\varepsilon + \frac{\varepsilon^2}{3}(3\xi^2-y^2)\right] f(\xi)\, d\xi + 0\,(c_0^6).$$

Analogous expansions are introduced for the remaining members. Hence, by conserving the accuracy specified above, we replace the system (1.9)-(1.12) by the system

$$\int\limits_{-c}^{c} [f_1+(x-\xi) f_1'] H_1(x-\xi)\, d\xi - \int\limits_{-a}^{a} \{(2f_3+\eta f_3') h_1(x,\eta) + (f_4+\eta f_4') h_2(x,\eta)\}\, d\eta = -\pi\sigma_2; \quad (1.13)$$

$$\int\limits_{-c}^{c} f_2 H_1(x-\xi)\, d\xi + \int\limits_{-a}^{a} \{\eta f_4' h_1(x,\eta) - (f_3+\eta f_3') h_2(x,\eta)\}\, d\eta = \pi\tau; \quad (1.14)$$

$$\int\limits_{-a}^{a} [f_4+(y-\eta) f_4'] H_2(y-\eta)\, d\eta + \int\limits_{-c}^{c} \{(2f_2+yf_1') h_2(\xi,y) - (f_1-yf_2') h_1(\xi,y)\}\, d\xi = -\pi\sigma_1; \quad (1.15)$$

$$\int\limits_{-a}^{a} f_3 H_3(y-\eta)\, d\eta + \int\limits_{-c}^{c} \{yf_2' h_2(\xi,y) - (f_2+yf_1') h_1(\xi,y)\}\, d\xi = \pi\tau. \quad (1.16)$$

The kernels of Eqs. (1.13)-(1.16) are

$$H_1(t) = \frac{1}{t} - \frac{\varepsilon}{3} t - \frac{\varepsilon^2}{45} t^3, \quad H_2(t) = \frac{1}{t} + \frac{\varepsilon}{3} t - \frac{\varepsilon^2}{45} t^3,$$

$$H_3(t) = \frac{1}{t} - \frac{\varepsilon}{3} t + \frac{\varepsilon^2}{15} t^3,$$

$$h_1(s,t) = s\left[\varepsilon - \frac{\varepsilon^2}{3}(3t^2-s^2)\right], \quad h_2(s,t) = t\left[\varepsilon - \frac{\varepsilon^2}{3}(3s^2-t^2)\right].$$

Let us note that for $a = 0$ the system (1.9)-(1.12) degenerates into

$$\frac{1}{\pi}\int_{L'}\frac{f_1(\xi)\,d\xi}{x-\xi} = -\sigma_2; \quad \frac{1}{\pi}\int_{L'}\frac{f_2(\xi)\,d\xi}{x-\xi} = \tau,$$

whose solution yields results obtained for a number of collinear cracks [3]. For $c = 0$, $\sigma_1 = \sigma_2 = 0$, $f_4 = 0$ and conservation of $(a/b)^2$ accuracy in the solution, the system (1.13)-(1.16) reduces to the equation

$$\int_{-a}^{a}\frac{f_3(\eta)\,d\eta}{y-\eta} - \frac{\pi^2}{12b^2}\int_{-a}^{a} f_3(\eta)(y-\eta)\,d\eta = \pi\tau$$

whose solution turns out to agree with the solution obtained in [4] for a number of parallel cracks.

2. Let $\sigma_1 = \sigma_2 = 0$, $\tau \neq 0$. Let us assume $f_1(\xi) = f_4(\eta) = 0$ and the functions $f_2(\xi)$ and $f_3(\eta)$ to be odd relative to their arguments. Equations (1.13) and (1.15) will automatically be satisfied, but after integration by parts (1.14) and (1.16) are written as

$$\int_{-c}^{c} f_2(\xi) H_1(x-\xi) + \int_{-a}^{a} f_3(\eta)\,\eta\,[\varepsilon + \varepsilon^2(x^2-\eta^2)]\,d\eta = \pi\tau;$$

$$\int_{-a}^{a} f_3(\eta) H_3(y-\eta)\,d\eta - \int_{-c}^{c} f_2(\xi)\left[\varepsilon + \frac{\varepsilon^2}{3}(3y^2+\xi^2)\right]d\xi = \pi\tau.$$

Let us make the substitution

$$\xi = -c\cos\varphi, \quad x = -c\cos\varphi_0, \quad \eta = -a\cos\vartheta, \quad y = -a\cos\vartheta_0$$

and let us assume

$$f_2(\xi) = \frac{\tau c\omega_2(\xi)}{\sqrt{c^2-\xi^2}}, \quad f_3(\eta) = \frac{\tau a\omega_3(\eta)}{\sqrt{a^2-\eta^2}},$$

where

$$\omega_2(\xi) = \omega_2(-c\cos\varphi) = \sum_{n=1}^{\infty}\omega_{2n}\cos(2n+1)\varphi, \tag{2.1}$$

$$\omega_3(\eta) = \omega_3(-a\cos\vartheta) = \sum_{n=1}^{\infty}\omega_{3n}\cos(2n+1)\vartheta.$$

Using the formula

$$\int_0^{\pi}\frac{\cos n\alpha\,d\alpha}{\cos\alpha-\cos\beta} = \pi\frac{\sin n\beta}{\sin\beta},$$

we obtain an algebraic system of equations to determine the coefficients of the expansions (2.1). Using the notation $p = (\pi c/b)^2$, $q = (\pi a/b)^2$ and conserving terms containing the quantities p and q in powers not above the second, we find the coefficients of the desired functions in the case of problems with shear:

$$\omega_{20} = 1 + \frac{p}{24} + \frac{q}{8} + \frac{p^2}{360} - \frac{pq-7q^2}{384}, \quad \omega_{21} = \frac{p^2}{1920} + \frac{pq}{128}, \tag{2.2}$$

$$\omega_{30} = 1 - \frac{p}{8} + \frac{q}{24} - \frac{5p^2+11pq}{384} - \frac{q^2}{720}, \quad \omega_{31} = -\frac{pq}{128} - \frac{q^2}{640}.$$

Let $\sigma_1 \neq 0$, $\sigma_2 \neq 0$, $\tau = 0$. Let us assume $f_2(\xi) = f_3(\eta) = 0$ and functions $f_1(\xi)$, $f_4(\eta)$ to be odd. Repeating the same reasoning, in the case of longitudinal tension ($\sigma_1 \neq 0$, $\sigma_2 = \tau = 0$)

$$f_1(\xi)=\frac{\sigma_1 c\alpha_1(\xi)}{\sqrt{c^2-\xi^2}},\quad f_4(\eta)=\frac{\sigma_1 a\alpha_4(\eta)}{\sqrt{a^2-\eta^2}};\tag{2.3}$$

$$\alpha_1(\xi)=\alpha_1(-c\cos\varphi)=\sum_{n=1}^{\infty}\alpha_{1n}\cos(2n+1)\varphi,$$
$$\alpha_4(\eta)=\alpha_4(-a\cos\vartheta)=\sum_{n=1}^{\infty}\alpha_{4n}\cos(2n+1)\vartheta,\tag{2.4}$$

where

$$\alpha_{10}=-\frac{q}{8}+\frac{5}{128}q^2-\frac{3}{128}pq,\quad \alpha_{11}=-\frac{pq}{128},$$
$$\alpha_{40}=-1-\frac{q}{8}-\frac{q^2}{48}+\frac{pq}{64},\quad \alpha_{41}=-\frac{q^2}{334}.\tag{2.5}$$

In exactly the same way in the case of transverse tension ($\sigma_1=\tau=0$, $\sigma_2\neq 0$), by replacing the functions $\alpha_1(\xi)$, $\alpha_4(\eta)$ in (2.3) and (2.4) by $\beta_1(\xi)$, $\beta_4(\eta)$ and the coefficients $\alpha_{jn}$ by $\beta_{jn}$, we find

$$\beta_{10}=-1-\frac{p}{8}-\frac{p^2}{48}+\frac{pq}{64},\quad \beta_{11}=-\frac{p^2}{334},$$
$$\beta_{40}=\frac{p}{8}+\frac{3p^2}{128}-\frac{pq}{128},\quad \beta_{41}=-\frac{pq}{128}.\tag{2.6}$$

Let us evaluate the value of the stress-intensity coefficients at the ends of the longitudinal and transverse slits. To determine them, as is usual [5], let us introduce the functions $Z_1(z)$, $Z_2(z)$ associated, respectively, with the tension $\sigma$ and the shear $\tau$, whose argument is the complex variable $z = x + iy$ ($\bar{z} = x - iy$). Hence, in the tension case

$$\sigma_x=\operatorname{Re}Z_1-y\operatorname{Im}Z_1',\quad \sigma_y=\operatorname{Re}Z_1+yZ_1'$$

and in the shear case

$$\sigma_x=2\operatorname{Im}Z_2+y\operatorname{Re}Z_2',\quad \sigma_y=-y\operatorname{Re}Z_2'.\tag{2.7}$$

By virtue of the relationships (2.7) and the representations (1.5)-(1.8) we have

$$2(\operatorname{Re}Z_1+\operatorname{Im}Z_2)=\sigma_x+\sigma_y=\Delta U=\frac{2}{\pi}\int_{L'}\frac{(x-\xi)f_1+yf_2}{(x-\xi)^2+y^2}d\xi+$$
$$+\sum_{k=-\infty}^{\infty}\frac{2}{\pi}\int\frac{(x-b_k)f_3+(y-\eta)f_4}{(x-b_k)^2+(y-\eta)^2}d\eta=2\operatorname{Re}\frac{1}{\pi}\int_{L'}\frac{f_1+if_2}{x-\xi+iy}d\xi-$$
$$-2\operatorname{Im}\sum_{k=-\infty}^{\infty}\frac{1}{\pi}\int_{L_n}\frac{f_4-if_3}{x-b_k-iy-i\eta}d\eta.$$

Therefore,

$$Z_1(z)=\frac{1}{\pi}\int_{L'}\frac{f_1d\xi}{z-\xi}+\sum_{k=-\infty}^{\infty}\frac{1}{\pi}\int_{L_k}\frac{f_4d\eta}{z-b_k-i\eta};$$
$$Z_2(z)=\frac{1}{\pi}\int_{L'}\frac{f_2d\xi}{z-\xi}+\sum_{k=-\infty}^{\infty}\frac{1}{\pi}\int_{L_k}\frac{f_3d\eta}{z-b_k-i\eta}.\tag{2.8}$$

Extracting the singularity at the ends $z = \pm c$ in the first of these expressions, we can write

$$Z_1(z)=\frac{1}{\pi}\int_{-c}^{c}\frac{f_1(\xi)d\xi}{z-\xi}+S_1(z),\tag{2.9}$$

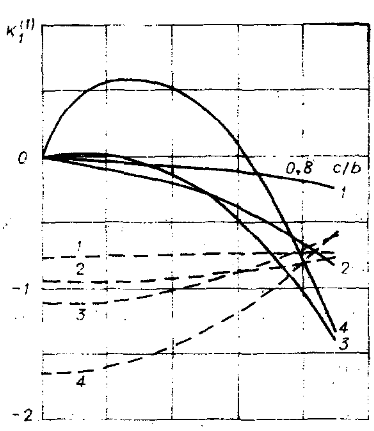


Fig. 2

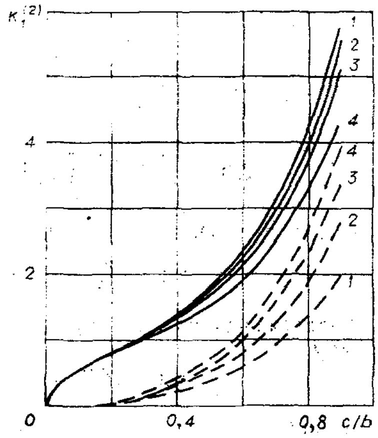


Fig. 3

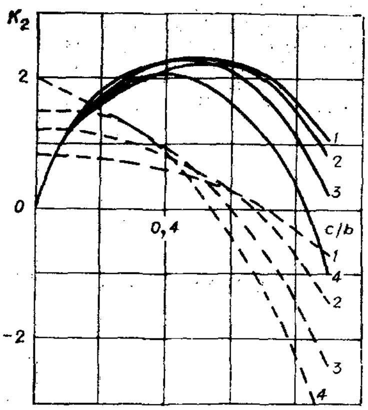


Fig. 4

where $S_1(z)$ is a regular function in the neighborhood of a longitudinal slit. By virtue of (2.3) and (2.4), for longitudinal tension

$$\frac{1}{\pi}\int_{-c}^{c}\frac{f_1(\xi)\,d\xi}{z-\xi}=\frac{1}{\pi}\int_{-c}^{c}\frac{\sigma_1\left[\alpha_{10}\xi+\frac{\alpha_{11}}{c^2}(4\xi^3-3\xi c^2)\right]}{\sqrt{c^2-\xi^2}\,(z-\xi)}\,d\xi=$$

$$=\frac{\sigma(\alpha_{10}+\alpha_{11})}{\pi}\int_{-c}^{c}\frac{\xi d\xi}{\sqrt{c^2-\xi^2}\,(z-\xi)}-\frac{4\sigma_1\alpha_{11}}{\pi c^2}\int_{-c}^{c}\frac{\xi\sqrt{c^2-\xi^2}}{\xi-z}\,d\xi=$$

$$=\sigma_1\left[(\alpha_{10}+\alpha_{11})\left(\frac{z}{\sqrt{z^2-c^2}}-1\right)+\frac{4\alpha_{11}}{c^2}\left(z\sqrt{z^2-c^2}-z^2-\frac{c^2}{2}\right)\right]. \qquad (2.10)$$

Evaluation of the integrals encountered here is presented in [6].

Therefore, the singularity of the function $Z_1(z)$ is $\sigma(\alpha_{10}+\alpha_{11})\times z/\sqrt{z^2-c^2}$, whereupon the stress-intensity factor at the end of a longitudinal slit under longitudinal tension turns out to equal

$$K_{1c}^{(1)}=\lim_{x\to c+0}\left[\sqrt{2\pi(x-c)}\,Z_1(x,0)\right]=\lim_{x\to c}\sqrt{2\pi(x-c)}\,\sigma_1(\alpha_{10}+\alpha_{11})\frac{x}{\sqrt{x^2-c^2}}=\sigma_1\sqrt{\pi c}\,(\alpha_{10}+\alpha_{11}). \qquad (2.11)$$

The following system of notation is taken here for the stress-intensity factor: the subscript 1 indicates tension and, hence, the superscripts 1 or 2 correspond to tension in the longitudinal and transverse directions; the subscript 2 corresponds to shear, while the subscripts c and $a$ correspond to the longitudinal and transverse slits.

Analogously, we obtain

$$K_{2c}=\lim_{x\to c+0}\left[\sqrt{2\pi(x-c)}\,Z_2(x,0)\right]=\tau\sqrt{\pi c}\,(\omega_{10}+\omega_{11}).$$

For transverse tension $K_{1c}^{(2)}$, $\sigma_2$, $\beta_{10}$, $\beta_{11}$ should be written, respectively, instead of $K_{1c}^{(1)}$, $\sigma_1$, $\alpha_{10}$, $\alpha_{11}$.

The values of these same quantities at the end of the transverse slit ($x = b$, $y = \pm a$) are obtained by replacing c by $a$ and the subscripts 10, 11, 20, 21 on the quantities $\alpha$, $\beta$, $\omega$ by 40, 41, 30, 31, respectively. The quantities $\alpha_{mn}$, $\beta_{mn}$, $\omega_{mn}$ are hence determined by the relationships (2.2), (2.5), and (2.6).

The coefficients $K_1^{(1)}/\sigma_1\sqrt{b}$, $K_1^{(2)}/\sigma_2\sqrt{b}$, $K_2/\tau\sqrt{b}$ are represented in Figs. 2-4 as a function of c/b. Curves 1-4 correspond to values of $a$/b equal to 0.2, 0.4, 0.6, 0.8. The solid lines show values of the intensity factors at the end of the longitudinal slit ($x = c$), and the dashes show the values at the end of the transverse slit ($y = a$). The singularity in Fig. 2 is the presence of a quite definite maximum of the coefficient $K_{1c}^{(1)}$ in the case of longitudinal tension for high values of $a$/b. At the same time, under shear (Fig. 4) all the curves $K_{2c}$ have maximal values, where this maximum shifts towards lower values of c/b with the increase in $a$/b. The behavior of $K_{2a}$ is analogous as $a$/b changes.

Finally, let us calculate the elastic strain energy increment on a crack (the work of opening during crack formation). The transverse displacements of the edge of a longitudinal slit $L_0'$ during tension are determined by the relationship

$$2Gv_j = \frac{\varkappa + 1}{2} \operatorname{Im} \widetilde{Z}_1^{(j)} - y \operatorname{Re} Z_1^{(j)} \quad (j = 1, 2),$$

and its longitudinal displacements under shear are

$$2Gu = \frac{\varkappa + 1}{2} \operatorname{Im} \widetilde{Z}_2 + y \operatorname{Re} Z_2.$$

Here $\varkappa = (3-\nu)/(1+\nu)$; $2G = E/(1+\nu)$ is Young's modulus, $\nu$ is the Poisson ratio, the functions $Z_1(z)$, $Z_2(z)$ are determined by the equalities (2.8), and $\widetilde{Z}_1$, $\widetilde{Z}_2$ are their primitives.

The energy increment on the slit $L_0'$ per unit thickness of the sheet in which the crack formed will be written as

$$\Delta A = \frac{1}{2} \int_{-c}^{c} \left[\sigma_2 (v_1^+ - v_1^-) + \sigma_2 (v_2^+ - v_2^-) + \tau (u^+ - u^-)\right] dx.$$

Let us evaluate the first member. Integrating the function $Z_1^{(1)}$ represented by (2.9) and (2.10) yields

$$\widetilde{Z}_1^{(1)}(z) = \sigma_1 \left[(\alpha_{10} + \alpha_{11}) \left(\sqrt{z^2 - c^2} - z\right) + \frac{4\alpha_{11}}{3c^2} \left(\sqrt{(z^2 - c^2)^3} - z^3 - \frac{3}{2} c^2 z\right) + \widetilde{S}_1(z)\right],$$

where $\widetilde{S}_1(z)$ is a regular function on $L_0'$.

Therefore, on a longitudinal slit

$$2G(v_1^+ - v_1^-) = (\varkappa + 1) \operatorname{Im} \widetilde{Z}_1^+;$$

$$\operatorname{Im} \widetilde{Z}_1^{(1)+} = \sigma_1 \left[(\alpha_{10} + \alpha_{11}) \sqrt{c^2 - x^2} - \frac{4}{3c^2} \alpha_{11} \sqrt{(c^2 - x^2)^3}\right];$$

$$\Delta A_1 = \frac{1}{2G} \frac{\varkappa + 1}{2} \int_{-c}^{c} \sigma_1 \operatorname{Im} \widetilde{Z}_1^{(1)+}(x)\, dx = \frac{\pi c^2}{E} \sigma_1^2 \alpha_{10}.$$

Finally, the energy increment on one longitudinal crack becomes

$$\Delta A_c = \frac{\pi c^2}{E} \left(\sigma_1 \sigma_2 \alpha_{10} + \sigma_2^2 \beta_{10} + \tau^2 \omega_{20}\right).$$

We obtain the energy increment on a transverse crack analogously,

$$\Delta A_a = \frac{\pi a^2}{E} \left(\sigma_1^2 \alpha_{40} + \sigma_1 \sigma_2 \beta_{40} + \tau^2 \omega_{30}\right),$$

where the quantities $\alpha_{n_0}$, $\beta_{n_0}$, $\omega_{n_0}$ are determined by (2.2) (2.5), and (2.6).

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